Q No.- Verify Rolle's theorem over $[a, b]$ for the function $f(x) = (x-a)^m (x-b)^n$,

m & n being a positive integer and find a suitable point c.

Dr. Sunil Jee
S.B. College, Ara

Soln.:- $\because$ ① $f(x) = (x-a)^m (x-b)^n$

Hence, we see that ① $f(x)$ is continuous in closed interval $[a, b]$.

② differentiable in $]a, b[$

③ $f(a) = f(b) = 0$

$\because f(x) = (x-a)^m (x-b)^n$

$\therefore f'(x) = m(x-a)^{m-1} \cdot (x-b)^n + n(x-b)^{n-1} \cdot (x-a)^m$

Putting $f'(x) = 0$.

$(x-a)^{m-1} (x-b)^{n-1} [m(x-b) + n(x-a)] = 0$.

Either $x = a$, Either $x = b$.

or, $mx - mb + nx - na = 0$

or, $x(m+n) = mb + na$

$\therefore x = \dfrac{mb+na}{m+n}$

Hence, the values $\dfrac{mb+na}{m+n}$ lies in $]a, b[$

$\therefore c = \dfrac{mb+na}{m+n}$ in open interval $]a, b[$

for which $f'(c) = 0$.

Hence, all condition of Rolle's theom. are satisfied.

Thus Rolle's theom. is verified.

② Q No.- Verify Rolle's theorem in the case of unction.

$f(x) = 2x^3 + x^2 - 4x - 2$.

Soln:- $\because f(x) = 2x^3 + x^2 - 4x - 2$.

As f is a rational integers function of x Hence it is Continuous and differentiable for all values of x. Hence Ist two Conditions of Rolle's theom is satisfied in any interval.

Now, $f(x) = 0$ given.

$$2x^3 + x^2 - 4x - 2 = 0$$

which on factorization

$$(x^2 - 2)(2x + 1) = 0$$

Either, $x^2 - 2 = 0$

$$x^2 = 2.$$

$$x = \pm\sqrt{2}$$

Either, $2x + 1 = 0$.

$$x = -\frac{1}{2}.$$

Hence, $f(\sqrt{2}) = f(-\sqrt{2}) = f(-\frac{1}{2}) = 0$.

Here, we shall take an interval $(-\sqrt{2}, \sqrt{2})$.

In order to verify Rolle's theom we have to show $f'(x) = 0$. at least once in the open interval $]-\sqrt{2}, \sqrt{2}[$.

$\because f'(x) = 6x^2 + 2x - 4$

which vanishes.

$$6x^2 + 2x - 4 = 0$$

$$3x^2 + x - 2 = 0$$

or, $3x^2 + 3x - 2x - 2 = 0$.

or, $3x(x+1) - 2(x+1) = 0$

or, $(x+1)(3x-2) = 0$.

Either, $x+1=0$

$x=-1$

$x=\frac{2}{3}$.

So that $f'(-1)=f'\left(\frac{2}{3}\right)=0$.

As the Points $x=-1$ and $x=\frac{2}{3}$ both lies in the $]-\sqrt{2},\sqrt{2}[$.

In this way Rolle's theorm is verified.